A thermodynamically consistent framework for saturated viscoplastic rock-materials subject to damage

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Abstract: The aim of this study is to build a thermodynamically consistent theoretical framework to model viscoplasticity and damage in saturated geomaterials. The induced anisotropic damage is represented by a second-order tensor. The key point of the model formulation is the definition of a "*double effective stress*", stemming from the concept of "*damaged effective stress*" (used in Continuum Damage Mechanics to couple damage and viscoplasticity in solids), and from the concept of "*hydro-mechanical effective stress*" (used in poromechanics to account for solid-fluid interactions). The dissipation potential is expressed as a function of the "*double effective stress*", which makes it possible to couple creep, fluid flow and damage in a consistent framework.

Keywords: thermodynamics; viscoplasticity; damage; poromechanics; couplings; effective stress

1. Introduction

Geomaterials such as rocks have a complex mechanical behaviour. Deformations can continue at fixed loads, failure can be brittle or ductile depending on loading conditions, and the behavior can be severely impacted by the presence of in-pore fluid. To start with, consider the case of salt-rock (Hunsche & Hampel, 1999 ; Hou, 2003). This material exhibits a remarkable time-dependent creep behavior, which has important practical implications. This can for example lead to a substantial increase of ground pressure on the lining support of a gallery. In practice, creep behavior is often simulated using viscoelastic or viscoplastic constitutive models. In other circumstances, damage phenomenon is crucial for the design of underground tunnels around which an area of "damaged" rock develops with time. At the microscopic scale, damage corresponds to the appearance and the evolution of micro-cracks, which induce a degradation of the stiffness and strength of a rock mass, as well as its hydraulic conductivity. This zone of damaged rock, often known as the Excavation Damaged Zone (EDZ), is of particular importance for disposal structures where the host rock plays a confining role (Tsang et al., 2005). In the domain of geomechanics, such behaviors are often studied using the approach of Continuum Damage Mechanics (CDM). In addition to creep of rock matrix, hydromechanical coupling between the latter and pore-water also lead to complex non linear time-dependent behaviors. Due to difficulties to simulate in a single model all these phenomena and their various couplings, simplifications are often adopted depending on the geotechnical conditions and the focus of the analysis. The modeling of anisotropic damage using CDM approach and coupled to other physical phenomena such as plasticity, viscoplasticity and/or hydromechanical interactions, appears to be a particularly difficult task.

At present, most existing constitutive laws do not account for the delayed damageable behavior, the only exception being Lemaitre's model (Lemaitre & Chaboche, 2001) which does not however account for hydromechanical coupling. Other developments dealt with the degradation of elastic (Lemaître & Desmorat, 2005) or poroelastic properties (Shao, 1998; Arson and Gatmiri, 2011), neglecting irreversible strains. Abu Al Rub & Voyiadjis (2003) and Hansen & Schreyer (1994) proposed a general thermodynamic framework to couple plasticity and damage in solids. Zhou et al. (2008) studied the coupling between damage, plasticity and viscoplasticity in monophasic porous media. In the same way, Chiarelli et al. (2003) created a model for plastic brittle rock-like materials, but the theoretical framework has only been developed in dry conditions. Conil *et al* (2004) and Maleki & Pouya (2010) studied saturated plastic rocks subject to cracking, in the absence of creep strains.

Some other interesting contributions using slightly different approaches have also been proposed by the community of geophysicists. Due to limited space, only a few of them are mentioned here. The work of Mckenzie (1984) considered a partially molten rock with melt-saturated porosities as a mixture of two viscous fluids with different viscosities, to study their compaction kinetics. The same principal hypothesis was also employed by Bercovici *et al* (2001) who used the classic averaging technique to obtain the macroscopic equations, and took also into account interface energies. However, at variance with the theory of continuum damage mechanics, they define "damage" as the part of the deformation work not dissipated but instead is converted into surface energy. While the assimilation of rock matrix as a viscous fluid (of a similar nature to the in-pore fluid) seems to be appropriate for problems involving geological time-scales, geotechnical engineers are more concerned with interactions between excavation-induced damage and hydro-mechanical behavior for a duration varying from tenths to hundreds of years. At this time-scale, CDM-type models coupled to viscoplastic creep would appear better adapted, and in any case more familiar to geotechnical engineers. The work of Hamiel *et al* (2004) is more in phase with this line of approach. They used a thermodynamic framework to build a model coupling damage to fluid flow and porosity evolution in a "poroelastic media". Actually, their model contains a viscous strain rate, of viscoelastic type, to account for creep, while damage is represented by a scalar, hence intrinsically limited to isotropic damage. Notwithstanding, one of the most interesting results of their work was on the antagonistic roles between pores and cracks: cracks act as stress concentrators promoting brittle failure, while pores dissipate stress concentrations leading to distributed deformations.

To the best of our knowledge, simultaneous couplings between viscoplastic creep, anisotropic damage and hydromechanical interactions have been insufficiently addressed. The closest related studies are focused on plasticity, with no account of creep effects. An original modeling approach is presented in this paper. The concept of "*double effective stress*" is introduced to account for damage and creep effects as well as poro-mechanical couplings.

Section 2 recalls the modeling objectives and presents the thermodynamic framework, mainly the free energy of the solid skeleton and the dissipation potential. Three elementary cases taking partial couplings into account are then reviewed: poro-elastic behaviour coupled with damage but without viscoplasticity (Section 3), viscoplastic behaviour coupled with damage in a monophasic porous medium (Section 4) and poro-visco-plastic behaviour without damage (Section 5). Finally, section 6 presents the constitutive equations of a fully coupled formulation, based on the concept of "*double effective stress*" and extending the partially coupled cases previously reviewed.

2. General thermodynamic framework

The aim of this paper is to formulate a realistic constitutive model for porous biphasic geomaterials like rocks, accounting for hydro-mechanical couplings and irreversible microstructure changes such as creep, hardening and damage. The porous medium is modelled using the poromechanics framework established by Coussy (1995, 2004), which presents an excellent synthesis of previous works due to Biot (1941, 1962), Berryman J.G. (1980), Detournay & Chung (1993)…, where the solid skeleton and the saturating pore fluid are considered as two superposed continua occupying the same geometrical space. Each of them has an independent velocity field, denoted by $\boldsymbol{v}^s$ (solid) and $\boldsymbol{v}^w$ (pore-fluid). The macroscopic description requires the knowledge of the volume fraction of each phase. In the sequel, the symbol ϕ will denote the volume fraction of the pore fluid, so that the fluid mass inside an elementary volume $d\Omega_t$ is given by $\rho_w \phi d\Omega_t$ and the mass of solid skeleton by $\rho_s(1-\phi)d\Omega_t$, where ρ_s and ρ_w are the intrinsic densities of the solid phase and the fluid phase, respectively.

For the solid skeleton, isothermal evolution and small perturbations will be assumed ; the latter assumption allows to consider the Eulerian and Lagrangian descriptions as equivalent. On the other hand, the fluid can enter and exit the physical volume considered, so that it is impossible to define a reference state based on the location of fluid particles. As a result, the position of solid particles is used to define the reference state, i.e. the Eulerian description is adopted. Hence, by construction, all physical quantities are made to depend on the initial coordinates of the solid skeleton. Strain and porosity changes of the solid phase will be assumed to admit a partition into a reversible part $\boldsymbol{\varepsilon}^e$ (resp. ϕ^e) and an irreversible part $\boldsymbol{\varepsilon}^{vp}$ (resp. ϕ^{vp}), that is to say $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^{vp}$ and $\phi - \phi_0 = \phi^e + \phi^{vp}$. The subscript $_0$ refers to the initial value (in the reference state). Moreover, the material in the initial state is assumed to be isotropic, with null initial pore pressure, stresses and strains. This initial state is also considered as the reference undamaged state used to set the initial value of the damage tensor. This second order tensor, considered as an internal state variable, can be used to model the damage "effects" in terms of stiffness and strength reduction as well as permeability increase due to the appearance and growth of microcracks in a continuous media setting.

The combination of the first and the second principles of thermodynamics (Coussy, 2004) yields the Clausius-Duhem inequality. Under isothermal condition, this writes:

$$\Phi_M + \Phi_T \geq 0 \quad \text{with } \Phi_M = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \mu^w div(\boldsymbol{w}) - \dot{\Psi}\,; \;\; \Phi_T = -\boldsymbol{w} \cdot \boldsymbol{grad}(\mu^w) \qquad (1)$$

Φ_M (respectively Φ_T) denotes the dissipation associated to mechanical (respectively transport) phenomena while Ψ is the total Helmholtz free energy per unit overall volume. μ^w and $\boldsymbol{w}$ are respectively the chemical potential and the mass flux vector of the pore fluid relative to the solid skeleton. Using classic arguments of independent dissipative mechanisms, each of the two components Φ_M and Φ_T is required to be independently non-negative. The non negativity of Φ_T leads to the classic Darcy law on pore fluid transport, which takes the following form under isothermal condition and the absence of body force:

$$\boldsymbol{w} = -\,\boldsymbol{k}_w.\boldsymbol{grad}(p_w) \qquad (2)$$

$\boldsymbol{k}_w$ is the permeability of the pore fluid. To recast the mechanical dissipation into the desired form, we firstly invoke the fluid mass conservation equation $div(\boldsymbol{w}) = -\rho_w \dot{\phi} - \dot{\rho}_w \phi$. Secondly, Ψ is expressed as the sum of the contributions from the solid skeleton and the pore fluid: $\Psi = \Psi_s + \rho_w \phi \Psi^w$. Here Ψ_s is the Helmholtz free energy of the solid skeleton per unit overall volume whereas Ψ^w is the specific Helmholtz free energy per unit mass of pore fluid. Note that the potentials Ψ^w and μ^w are linked by the relation $\mu^w - \Psi^w = p_w / \rho_w$. In parallel, the classic state equation of a barotropic fluid $d\Psi^w = -p_w d(1/\rho_w) - s_w dT$ implies $\rho_w \dot{\Psi}^w = p_w \dot{\rho}_w / \rho_w$ under isothermal condition. On account of the last four relations and the partition of strains and porosity variation, the mechanical dissipation Φ_M simplifies to the following form:

$$\Phi_M = \boldsymbol{\sigma} : (\dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^{vp}) + p_w(\dot{\phi}^e + \dot{\phi}^{vp}) - \dot{\Psi}_s \qquad (3)$$

In the case of monophasic media (Lemaitre & Chaboche (2001)), the skeleton free energy Ψ_s depends on the elastic strain $\boldsymbol{\varepsilon}^e$, the damage tensor $\boldsymbol{D}$ and hardening parameters like the cumulated equivalent viscoplastic strain γ_{vp} defined by $\gamma_{vp} = \int_0^t \sqrt{2/3\, \dot{\boldsymbol{\varepsilon}}^{vp} : \dot{\boldsymbol{\varepsilon}}^{vp}}\, dt$.

In the present case, the skeleton free energy Ψ_s also depends on the reversible porosity change ϕ^e. In other words, $\Psi_s = \Psi_s(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, \phi^e, \gamma_{vp})$. It is however more practical to work with the pore pressure p_w rather than with ϕ^e. A partial Lengendre-transform is then used to replace Ψ_s by a more appropriate thermodynamic potential, expressed as:

$$\Psi_s^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, p_w, \gamma_{vp}) = \Psi_s(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, \phi^e, \gamma_{vp}) - p_w \phi^e \qquad (4)$$

Combining equations Eqs.(3) and (4) , we get:

$$\Phi_M = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{vp} + \left(\boldsymbol{\sigma} - \frac{\partial \Psi_s^*}{\partial \boldsymbol{\varepsilon}^e}\right) : \dot{\boldsymbol{\varepsilon}}^e + p_w \dot{\phi}^{vp} - \left(\phi^e + \frac{\partial \Psi_s^*}{\partial p_w}\right)\dot{p}_w + k\dot{\gamma}_{vp} + \boldsymbol{Y} : \dot{\boldsymbol{D}} \geq 0 \qquad (5)$$

where k and $\boldsymbol{Y}$ are the thermodynamic forces conjugate to γ_{vp} and $\boldsymbol{D}$, respectively. They are defined by:

$$Y = -\frac{\partial \Psi_s^*}{\partial D} \quad ; \quad k = -\frac{\partial \Psi_s^*}{\partial \gamma_{vp}} \tag{6}$$

During reversible evolutions, irreversible strains, damage and hardening variable all remain constant, while the dissipation is null. Noting that the variations of the elastic deformation and pore pressure are independent, the state equations write:

$$\boldsymbol{\sigma} = \frac{\partial \Psi_s^*}{\partial \boldsymbol{\varepsilon}^e} \quad ; \quad \phi^e = -\frac{\partial \Psi_s^*}{\partial p_w} \tag{7}$$

These equations are assumed to hold even during irreversible evolutions (Coussy 2004). On account of the last two state equations, the dissipation inequality becomes:

$$\Phi_M = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{vp} + p_w \dot{\phi}^{vp} + k \dot{\gamma}_{vp} + \boldsymbol{Y} : \dot{\boldsymbol{D}} \geq 0 \tag{8}$$

The classic approach to satisfy the above inequality is to postulate a dissipation potential $\varphi^*(\boldsymbol{\sigma}, p_w, k, \boldsymbol{Y} / \boldsymbol{D}, \gamma_{vp})$, positive-definite and convex relative to its principle arguments ($\boldsymbol{\sigma}, p_w, k, \boldsymbol{Y}$), verifying:

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \frac{\partial \varphi^*}{\partial \boldsymbol{\sigma}} \quad ; \quad \dot{\phi}^{vp} = \frac{\partial \varphi^*}{\partial p_w} \quad ; \quad \dot{\gamma}_{vp} = \frac{\partial \varphi^*}{\partial k} \quad ; \quad \dot{\boldsymbol{D}} = \frac{\partial \varphi^*}{\partial \boldsymbol{Y}} \tag{9}$$

Note that in the functional notation $\varphi^*(\boldsymbol{\sigma}, p_w, k, \boldsymbol{Y} / \boldsymbol{D}, \gamma_{vp})$, arguments that come before the sign "/" are considered as variables whereas those after the sign "/" are parameters (Lemaitre & Chaboche, 2001). The positive-definite and convex character of φ^* then allows to confirm the non-negativity of Φ_M:

$$\Phi_M = \boldsymbol{\sigma} : \frac{\partial \varphi^*}{\partial \boldsymbol{\sigma}} + p_w \frac{\partial \varphi^*}{\partial p_w} + k \frac{\partial \varphi^*}{\partial k} + \boldsymbol{Y} : \frac{\partial \varphi^*}{\partial \boldsymbol{Y}} \geq 0 \tag{10}$$

In principle, and only in principle, the constitutive model is entirely defined by the two thermodynamic potentials Ψ_s^* and φ^*. But in the most general case (where viscoplastic strains, fluid pressure and damage are simultaneously present), these functions are very difficult to determine, particularly their coupled dependence on pore pressure, damage tensor and hardening parameter.

To simplify, the first step is to assume the following expression of the free energy of the solid skeleton, in order to uncouple viscoplasticity from the other phenomena:

$$\Psi_s^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, p_w, \gamma_{vp}) = \Psi_1^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, p_w) + \Psi_2^*(\gamma_{vp}) \tag{11}$$

In addition, damage is isolated in the expression of the dissipation potential:

$$\varphi^*(\boldsymbol{\sigma}, p_w, k, \boldsymbol{Y} / \boldsymbol{D}, \gamma_{vp}) = \varphi_1^*(\boldsymbol{Y} / \boldsymbol{D}) + \varphi_2^*(\boldsymbol{\sigma}, p_w, k / \boldsymbol{D}, \gamma_{vp}) \tag{12}$$

Having done this, it remains to define the functional form of Ψ_s^* and that of the dissipation potential φ^*. A step-by-step modeling approach is adopted. The following particular couplings are studied separately before the formulation of the constitutive model coupling damage, viscoplasticity and solid-fluid interactions:

Case 1. Poro-elasticity coupled with damage ($\boldsymbol{\varepsilon}^{vp} = \boldsymbol{0}, \phi^{vp} = \gamma_{vp} = 0$)

Case 2. Viscoplasticity coupled with damage in a monophasic porous solid ($p_w = 0$)

Case 3. Poro-visco-plasticity ($\boldsymbol{D} = \boldsymbol{Y} = \boldsymbol{0}$)

In short, the review of case 1 allows to deduce the form of Ψ_1^*, which then allows to express explicitly the state equations Eq.(7). The introduction of a "*damaged effective stress*" and the *Principle of Equivalent Elastic Energy* (PEEE) then yield the explicit form for the damaged stiffness tensor and other elastic coefficients. To complete the modeling framework of an elastic damageable material, the form of the damage dissipation potential $\varphi_1^*(\boldsymbol{Y}/\boldsymbol{D})$ needs to be determined. The study of case 2 suggests a particular form for the dissipation potential $\varphi_2^*(\boldsymbol{\sigma}, p_w, k/\boldsymbol{D}, \gamma_{vp})$, and also a way to couple viscoplasticity with damage. The review of case 3 leads instead to the definition of an effective stress in the sense of Biot, to account for pore-pressure effects on the irreversible strains. These studies ultimately provide a formulation based on a "*double effective stress*", accounting simultaneously for damage and pore-pressure effects.

3. Case 1: Poro-elasticity coupled with damage ($\boldsymbol{\varepsilon}^{vp} = \boldsymbol{0}, \phi^{vp} = \gamma_{vp} = 0$)

In case 1, the potential Ψ_s^* is reduced to its elastic damageable part Ψ_1^*. Various forms of the elastic potential were used in previous papers to model poroelastic damageable behaviour. In particular, Maleki & Pouya (2010) and Conil et al. (2004) suggested two potentials respectively based on Dragon (2002) and Lemaitre & Desmorat (2005) works. Additional parameters are introduced in both elastic potentials to account for damage effects on elastic properties. Here, following Shao (1998), the damage elastic potential Ψ_1^* is assumed to be:

$$\Psi_1^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, p_w) = \tfrac{1}{2}\boldsymbol{\varepsilon}^e : \mathbf{C}(\boldsymbol{D}) : \boldsymbol{\varepsilon}^e - p_w \boldsymbol{B}(\boldsymbol{D}) : \boldsymbol{\varepsilon}^e - \tfrac{1}{2}\beta(\boldsymbol{D}) p_w^2 \qquad (13)$$

$\mathbf{C}(\boldsymbol{D}), \boldsymbol{B}(\boldsymbol{D})$ and $\beta(\boldsymbol{D})$ are the damaged rigidity, Biot modulus and the hydro-mechanical coupling parameter, respectively. The first term of this potential accounts for damage effects on the elastic stiffness whereas the two last terms considers the damage influence on the coupling coefficients. The combination of Eqs.(8) and (13) leads to the following explicit state equations:

$$\boldsymbol{\sigma} = \mathbf{C}(\boldsymbol{D}) : \boldsymbol{\varepsilon}^e - p_w \boldsymbol{B}(\boldsymbol{D}) \quad ; \quad \phi^e = \boldsymbol{B}(\boldsymbol{D}) : \boldsymbol{\varepsilon}^e + p_w \beta(\boldsymbol{D}) \qquad (14)$$

According to the continuum mechanics sign convention, compressive stresses are considered negative. The thermodynamic force $\boldsymbol{Y}$ is obtained by substitution of Eq.(4) into Eq.(6):

$$\boldsymbol{Y} = -\tfrac{1}{2}\boldsymbol{\varepsilon}^e : \frac{\partial \mathbf{C}(\boldsymbol{D})}{\partial \boldsymbol{D}} : \boldsymbol{\varepsilon}^e + p_w \frac{\partial \boldsymbol{B}(\boldsymbol{D})}{\partial \boldsymbol{D}} : \boldsymbol{\varepsilon}^e + \tfrac{1}{2}\frac{\partial \beta(\boldsymbol{D})}{\partial \boldsymbol{D}} p_w^2 \qquad (15)$$

3.1. Principle of Equivalent Elastic Energy

To determine the damage-dependent parameters $\mathbf{C}(\boldsymbol{D}), \boldsymbol{B}(\boldsymbol{D})$ and $\beta(\boldsymbol{D})$, the classic concepts of *Principle of Equivalent Elastic Energy* (PEEE) and "*damaged effective stress"* (DES) are introduced. Consider first a dry material or a saturated material under drained conditions (i.e. $p_w = 0$), the elastic potential Ψ_1^* then writes:

$$\Psi_1^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, 0) = 1/2\boldsymbol{\varepsilon}^e : \boldsymbol{C}(\boldsymbol{D}) : \boldsymbol{\varepsilon}^e \tag{16}$$

Gibbs free energy $G_s = G_s(\boldsymbol{\sigma}, \boldsymbol{D}, 0)$ depending on stress instead of strains, is obtained via a partial Legendre transform of $\Psi_1^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, 0)$:

$$G_s(\boldsymbol{\sigma}, \boldsymbol{D}, 0) = \Psi_1^*(\boldsymbol{\varepsilon}^e, \boldsymbol{D}, 0) - \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^e \tag{17}$$

The PEEE (Cordebois & Sidoroff, 1982; Hansen & Schreyer, 1994) states that the elastic potential of the damaged material subjected to the true stress is equal to the one of the undamaged material subjected to the DES $\tilde{\boldsymbol{\sigma}}$. In other words:

$$G_s(\boldsymbol{\sigma}, \boldsymbol{D}, 0) = G_s(\tilde{\boldsymbol{\sigma}}, \boldsymbol{D} = \boldsymbol{0}, 0) \tag{18}$$

The concept of damaged effective stress in Continuum Damage Mechanics allows to model the effects of stress concentration and reorientation induced by damage. It depends linearly on the classic stress tensor and is defined with an operator $\boldsymbol{M}$ that depends on the damage tensor but that is not related to the material constitutive behavior (Hansen & Schreyer, 1994)

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{M}(\boldsymbol{D}) : \boldsymbol{\sigma} \tag{19}$$

Various approaches have been adopted to define $\boldsymbol{M}(\boldsymbol{D})$ (Cordebois & Sidoroff, 1982; Murakami, 1988; Betten, 1986; Lu & Chow, 1991; Lemaitre & Desmorat, 2005; Maleki & Pouya, 2010) depending on the modelling approach and the definition of $\tilde{\boldsymbol{\sigma}}$. Our preference goes to Cordebois & Sidoroff (1982) operator which writes:

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{H}(\boldsymbol{D}) \cdot \boldsymbol{\sigma} \cdot \boldsymbol{H}(\boldsymbol{D}) \quad ; \quad \boldsymbol{H}(\boldsymbol{D}) = (\boldsymbol{\delta} - \boldsymbol{D})^{-1/2} \tag{20}$$

where $\boldsymbol{\delta}$ is the second order identity tensor. This leads to the following form of the fourth-order tensor $\boldsymbol{M}(\boldsymbol{D})$, which satisfies both major and minor symmetries:

$$\boldsymbol{M}_{ijkl} = \tfrac{1}{2}(H_{ik}H_{jl} + H_{il}H_{jk}) \tag{21}$$

Substituting Eq. (19) into Eq.(18) allows to determine the damaged stiffness tensor:

$$\boldsymbol{C}(\boldsymbol{D}) = \boldsymbol{M}^{-1}(\boldsymbol{D}) : \boldsymbol{C}_0 : \boldsymbol{M}^{-T}(\boldsymbol{D}) = \boldsymbol{M}^{-1}(\boldsymbol{D}) : \boldsymbol{C}_0 : \boldsymbol{M}^{-1}(\boldsymbol{D}) \tag{22}$$

where $\boldsymbol{C}_0$ is the stiffness tensor of the virgin material, assumed to be isotropic. It can be expressed with Lame's coefficients λ and μ: $\boldsymbol{C}_0 = \lambda\boldsymbol{\delta} \otimes \boldsymbol{\delta} + 2\mu\boldsymbol{I}$ with $\boldsymbol{I}$ being the fourth order identity tensor, with components $I_{ijkl} = 1/2(\boldsymbol{\delta}_{ik}\boldsymbol{\delta}_{jl} + \boldsymbol{\delta}_{il}\boldsymbol{\delta}_{jk})$. Eq.(22) shows that the damaged stiffness tensor satisfies both major and minor symmetries. Its computation only requires Lame's coefficients and the damage tensor, without any other additional parameters. The above expression of $\boldsymbol{C}(\boldsymbol{D})$ is assumed to remain valid in the more general case of non zero pore pressure $p_w \neq 0$.

Finally, previous analyses based on micro-poromechanics (Shao, 1998) suggested the following expressions of the damage-dependent hydro-mechanical parameters $\boldsymbol{B}(\boldsymbol{D})$ and $\beta(\boldsymbol{D})$:

$$\boldsymbol{B}(\boldsymbol{D}) = \boldsymbol{\delta} - \frac{1}{3K_s}\boldsymbol{C}\,(\boldsymbol{D}):\boldsymbol{\delta} \;\; ; \quad \beta(\boldsymbol{D}) = \frac{1}{K_s}\left(\frac{1}{3}\mathrm{tr}(\boldsymbol{B}(\boldsymbol{D})) - \phi_0\right) \tag{23}$$

where K_s is the compressibility of the solid constituent of the skeleton, which is to be distinguished from the compressibility of the solid skeleton, given by $\lambda + 2\mu/3$. At this stage, the only equation missing to complete the formulation of the poro-elastic damage model (case 1) is the damage evolution law. In this paper, Lemaitre's approach is adopted (Lemaitre & Chaboche, 2001).

3.2. Evolution law of induced anisotropic damage

In case 1, ($\boldsymbol{\varepsilon}^{vp} = \boldsymbol{0},\ \phi^{vp} = \gamma_{vp} = 0$), the Clausius-Duhem inequality Eq.(8) is reduced to:

$$\boldsymbol{Y} \cdot \dot{\boldsymbol{D}} > 0 \tag{24}$$

Various explicit forms of the damage dissipation potential φ_1^* exist in the literature. Mazars (1984) who studied the behaviour of concrete suggests a damage criterion depending on the extension strains. This approach was then followed by Shao (1998), Arson & Gatmiri (2011) and Conil et al. (2004). It has the advantage to link the damage evolution law to the strain in the material. However, this approach does not guarantee the positivity of the intrinsic dissipation.

More generally speaking, we can build a quadratic damage potential φ_1^* which is convex with respect to its variable $\boldsymbol{Y}$, and positive-definite. The following form is assumed:

$$\varphi_1^*(\boldsymbol{Y}/\boldsymbol{D}) = \frac{1}{2}F(\boldsymbol{D})\boldsymbol{Y} : \boldsymbol{S} : \boldsymbol{Y} \tag{25}$$

F is a scalar function depending on the damage variable whereas $\boldsymbol{S}$ is a dimensionless fourth-order tensor defining the damage evolution law, which is, taking Eq. (32) into account:

$$\dot{\boldsymbol{D}} = \frac{\partial \varphi_1^*}{\partial \boldsymbol{Y}} = F(\boldsymbol{D})\boldsymbol{S} : \boldsymbol{Y} \tag{26}$$

Note that $\dot{\boldsymbol{D}}$ has the dimension of the inverse of time (s^{-1}) while $\boldsymbol{Y}$ the dimension of stress ($N m^{-2}$). Hence to be dimensionally correct, φ_1^* should have the dimension of a stress rate ($N m^{-2} s^{-1}$). There are different ways to enforce this consistency, depending on the particular choice of the intervening functions. One simple way to achieve this is to impose the scalar function $F(\boldsymbol{D})$ to have the dimension of the inverse of the product of time and stress ($N^{-1} m^2 s^{-1}$) and the tensor $\boldsymbol{S}$

to be dimensionless. Pellet et al. (2005) suggests a particular form for the tensor $\boldsymbol{S}$:

$$\boldsymbol{S} = (\beta - 1)\boldsymbol{I} + \boldsymbol{\delta} \otimes \boldsymbol{\delta} \tag{27}$$

where β is a damage parameter bounded by 0 and 1. If $\beta = 0$ the induced damage is orthotropic. If $\beta = 1$ the induced damage is isotropic. As a matter of simplicity, $F(\boldsymbol{D})$ is chosen to be independent of the damage variable, that is to say: $F(\boldsymbol{D}) = F$, a constant. The combination of Eqs.(26) and (15) yields:

$$\dot{\boldsymbol{D}} = F\,\boldsymbol{S} : \left(-\tfrac{1}{2}\boldsymbol{\varepsilon}^e : \frac{\partial \boldsymbol{C}(\boldsymbol{D})}{\partial \boldsymbol{D}} : \boldsymbol{\varepsilon}^e + p_w \frac{\partial \boldsymbol{B}(\boldsymbol{D})}{\partial \boldsymbol{D}} : \boldsymbol{\varepsilon}^e + \tfrac{1}{2} \frac{\partial \beta(\boldsymbol{D})}{\partial \boldsymbol{D}} p_w^2 \right) \tag{28}$$

Because of the mathematical properties of the dissipation potential φ_1^*, the positivity of the mechanical dissipation(Eq.(8)) is automatically satisfied.

From now on, the expression of φ_1^* will no longer be discussed. In the absence of viscoplasticity, φ_1^* is determined by Eq.(25). The next challenge consists in expressing the viscoplastic potential so as to account for the hydro-mechanical and damage couplings.

4. Case 2: Viscoplasticity coupled with damage in a monophasic porous medium ($p_w = 0$)

In the sequel, a dry or saturated viscoplastic damageable material under drained conditions (i.e. $p_w = 0$) is considered. In case 2, the fundamental dissipation inequality (Eq.(8)) writes:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{vp} + k\dot{\gamma}_{vp} + \boldsymbol{Y} : \dot{\boldsymbol{D}} \geq 0 \tag{29}$$

The viscoplastic potential is still given by equation (12), with $\varphi^* = \varphi_1^* + \varphi_2^*$, where $\varphi_1^* = \varphi_1^*(\boldsymbol{Y}/\boldsymbol{D} = \boldsymbol{0})$ and $\varphi_2^* = \varphi_2^*(\boldsymbol{\sigma}, p_w, k/\boldsymbol{D} = \boldsymbol{0}, \gamma_{vp})$. φ_1^* is assumed to have the form Eq.(25) described in case 1. Using the procedure proposed by Lemaitre & Chaboche (2001) and followed by Pellet et al. (2005), the viscoplastic dissipation potential of a virgin material is defined from a load surface depending on the first and second stress invariants σ and q. Its expression is inspired from Lemaitre's dissipation potential $\varphi^*_{\mathscr{Q}}(\boldsymbol{\sigma}, k/\gamma_{vp})$:

$$\varphi_2^* = \varphi_2^*(\boldsymbol{\sigma}, p_w = 0, k/\boldsymbol{D} = \boldsymbol{0}, \gamma_{vp}) = \varphi^*_{\mathscr{Q}}(\boldsymbol{\sigma}, k/\gamma_{vp}) = \frac{K}{N+1} \left\langle \frac{q + \alpha\sigma - k}{K} \right\rangle^{N+1} \gamma_{vp}^{\;-N/M} \tag{30}$$

where $\sigma = 1/3\,tr(\boldsymbol{\sigma})$ is the mean stress, $\boldsymbol{s} = \boldsymbol{\sigma} - \sigma\boldsymbol{\delta}$ is the stress deviator and $q = \sqrt{3/2\boldsymbol{s} : \boldsymbol{s}}$ is the Von Mises equivalent stress. k defines the viscoplastic hardening, which expresses the fact that the threshold varies with the cumulated viscoplastic strains. The constant α characterises the influence of the mean stress on the viscoplastic volumetric strain. K, N and M are viscoplastic parameters. $\langle . \rangle$ are the Macaulay brackets, defined by $\langle x \rangle = 1/2(x + |x|)$.

According to the energy equivalence principle (Lemaitre et al, 2000; Cordebois & Sidoroff, 1982), the viscoplastic dissipation potential characterizing the behaviour of a damaged material can be determined by considering the viscoplastic potential of a sound material, whereby the real stress is replaced by the "*damaged effective stress*" $\tilde{\boldsymbol{\sigma}}$. In other words:

$$\varphi_2^*(\boldsymbol{\sigma}, 0, k/\boldsymbol{D}, \gamma_{vp}) = \varphi_2^*(\tilde{\boldsymbol{\sigma}}, 0, k/\boldsymbol{0}, \gamma_{vp}) \equiv \varphi^*_{\mathscr{Q}}(\tilde{\boldsymbol{\sigma}}, k/\gamma_{vp}) \tag{31}$$

The viscoplastic strain rate is obtained by applying the normality rule to the viscoplastic dissipation φ_2^*:

$$\dot{\varepsilon}^{vp} = \frac{\partial \varphi_2^*}{\partial \boldsymbol{\sigma}} = \frac{\partial \varphi_2^*}{\partial \tilde{\boldsymbol{\sigma}}} : \frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \boldsymbol{\sigma}} \tag{32}$$

The potential φ_2^* is convex with respect to its variables $\tilde{\boldsymbol{\sigma}}$ and k, positive and contains the origin. Consequently the positivity of the viscoplastic dissipation is ensured, that is to say:

$$\boldsymbol{\sigma} : \dot{\varepsilon}^{vp} + k\dot{\gamma}_{vp} \geq 0 \tag{33}$$

5. Case 3: Poro-visco-plasticity ($\boldsymbol{D} = \boldsymbol{Y} = \boldsymbol{0}$)

Considering a virgin saturated poroviscoplastic material Coussy (2004) highlighted the state equations for saturated viscoplastic materials in the absence of damage. The damaged rigidity $\boldsymbol{C}(\boldsymbol{D})$ and hydro-mechanical parameters $\boldsymbol{B}(\boldsymbol{D})$ and $\beta(\boldsymbol{D})$ (see Eqs.(22), (23)) are replaced by their reference values: $\boldsymbol{C}_0$, $\boldsymbol{B_0}$, β_0. The Clausius-Duhem inequality becomes:

$$\boldsymbol{\sigma} : \dot{\varepsilon}^{vp} + p_w \dot{\phi}^{vp} + k\dot{\gamma}_{vp} \geq 0 \tag{34}$$

In view of the difficult conjecture on the dependence of the dissipation function on pore-pressure, a linear relationship is assumed between the viscoplastic porosity rate $\dot{\phi}^{vp}$ and the viscoplastic strain rate $\dot{\varepsilon}^{vp}$ (Coussy, 2004):

$$\dot{\phi}^{vp} = \boldsymbol{B_0} : \dot{\varepsilon}^{vp} \tag{35}$$

The above assumption allows to recast the mechanical dissipation in the form:

$$\boldsymbol{\sigma}' : \dot{\varepsilon}^{vp} + k\dot{\gamma}_{vp} \geq 0 \tag{36}$$

depending on the "*hydro-mechanical effective stress*" $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p_w \boldsymbol{B_0}$ and the hardening variable. This "*hydro-mechanical effective stress*" is the thermodynamic force associated to the viscoplastic strain. $\boldsymbol{B_0} = \boldsymbol{\delta}$ restricts ourselves to the case of an incompressible viscoplastic matrix. Then, the "*hydro-mechanical effective stress"* is similar to Terzaghi's effective stress.

According to the general approach of thermodynamics, the viscoplastic dissipation potential can be formulated in terms of $\boldsymbol{\sigma}'$ and k, thermodynamically associated to the viscoplastic strain and to the hardening variable γ_{vp}, respectively, that is to say $\varphi_2^*(\boldsymbol{\sigma}, p_w, k/\boldsymbol{0}, \gamma_{vp}) \equiv \varphi_{\mathcal{Q}}^*(\boldsymbol{\sigma}', k/\gamma_{vp})$(see Eq.(37)). The viscoplastic strain rate is computed from:

$$\dot{\varepsilon}^{vp} = \frac{\partial \varphi_2^*}{\partial \boldsymbol{\sigma}'} \tag{30}$$

6. Poroviscoplastic damageable model

To our knowledge, no model has been proposed so far to couple viscoplasticity to poromechanical damage. The closest related studies are focused on plasticity, with no account of creep effects. Chiarelli et al. (2003) created a model for plastic brittle rock-like materials, but the theoretical framework has only been developed in dry conditions. Damage is assumed to be purely mechanical, and to evolve with tensile strains. This mechanical damage is coupled to plasticity by means of a plastic criterion $f(\sigma, q, \theta / \boldsymbol{D}, \gamma_p)$ and a plastic potential $g(\sigma, q, \theta / \boldsymbol{D}, \gamma_p)$, in which σ and q are respectively the mean stress and the Von Mises equivalent stress and θ is Lode's angle. γ_p is a hardening plastic variable. Damage comes into the picture by assuming that f and g have the same expressions as the ones that hold for sound materials, whereby the real equivalent Von Mises stress q is replaced by the net deviatoric stress $\hat{q} = \sqrt{3/2\, \boldsymbol{s} : \boldsymbol{L}(\boldsymbol{D}) : \boldsymbol{s}}$. In other words:

$$f(\sigma, q, \theta / \boldsymbol{D}, \gamma_p) = f(\sigma, \hat{q}, \theta / \boldsymbol{0}, \gamma_p) \text{ and } g(\sigma, q, \theta / \boldsymbol{D}, \gamma_p) = g(\sigma, \hat{q}, \theta / \boldsymbol{D}, \gamma_p) \qquad (38)$$

$\boldsymbol{L}(\boldsymbol{D})$ is a fourth–order operator depending on damage, and playing the same role as the $\boldsymbol{M}(\boldsymbol{D})$ operator used in Section 3 (Eq.(21)). It is defined by:

$$\boldsymbol{L}_{ijkl}(\boldsymbol{D}) = \tfrac{1}{2}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}) + \tfrac{1}{2} c_{pm}(\delta_{ik} D_{jl} + D_{ik}\delta_{jl} + \delta_{il} D_{jk} + D_{il}\delta_{jk}):$$

The constant c_{pm} characterizes the importance of the damage effect on the plastic flow. If $c_{pm} = 0$ then $\hat{q} = q$. When the value of c_{pm} increases, the material behaviour becomes more and more brittle. The proposed evolution laws are very similar to the ones in Drucker-Prager's plasticity model.

Conil et al. (2004) and Maleki & Pouya (2010) studied saturated plastic rocks subject to cracking. Their model is based on a phenomenological approach, in which the plastic and damage dissipation potentials are uncoupled. Like Chiarelli, Maleki resorts to a plastic criterion and a plastic potential that are based on a modified Drucker-Prager's yield function and a non associated flow rule and that couple the plastic variables with damage. In addition, $f(\sigma, q, \theta, p_w / \boldsymbol{D}, \gamma_p)$ and $g(\sigma, q, \theta, p_w / \boldsymbol{D}, \gamma_p)$ depend on the pore pressure. The plasticity criterion f and the plastic potential g are extended to damaged materials by replacing the real stress by the effective stress $\boldsymbol{\sigma}' = \boldsymbol{H}.\boldsymbol{s}.\boldsymbol{H} + \sigma\boldsymbol{\delta} + \boldsymbol{B}(\boldsymbol{D}) p_w$ in the expressions of f and g used for sound materials. In other words:

$$f(\sigma, q, \theta, p_w / \boldsymbol{D}, \gamma_p) = f(\sigma', q', \theta', 0 / \boldsymbol{0}, \gamma_p) \text{ and } g(\sigma, q, \theta, p_w / \boldsymbol{D}, \gamma_p) = g(\sigma', q', \theta', 0 / \boldsymbol{0}, \gamma_p) \qquad (39)$$

The same approach was followed by Conil. However, she states that the functions f and g of the damaged geomaterial can be determined by considering the functions f and g of a sound material, whereby the real equivalent stress is replaced by a "*damaged effective stress*" $\tilde{\boldsymbol{\sigma}} = \boldsymbol{H}.\boldsymbol{s}.\boldsymbol{H} - 1/3\, tr(\boldsymbol{H}.\boldsymbol{s}.\boldsymbol{H})\boldsymbol{\delta} + \sigma / \left(1 - \eta\, tr(\boldsymbol{D})\right)\boldsymbol{\delta}$ which did not take poro-mechanical coupling into account. In other words:

$$f(\sigma, q, \theta, p_w / \boldsymbol{D}, \gamma_p) = f(\tilde{\sigma}, \tilde{q}, \tilde{\theta}, 0 / \boldsymbol{0}, \gamma_p) \text{ and } g(\sigma, q, \theta, p_w / \boldsymbol{D}, \gamma_p) = g(\tilde{\sigma}, \tilde{q}, \tilde{\theta}, 0 / \boldsymbol{0}, \gamma_p) \qquad (40)$$

Having reviewed cases 1 to 3, we are now ready to tackle the general case considered in this paper, namely the coupled viscoplastic behaviour with anisotropic damage within the framework of saturated biphasic porous media.

Following the same approach as in the base case 3 (poro-viscoplasticity), it is assumed that the rate of change of the viscoplastic porosity is related to the viscoplastic deformation rate, as follows:

$$\dot{\tilde{\varepsilon}}^{vp} = \boldsymbol{B}(\boldsymbol{D}):\dot{\varepsilon}^{vp} \tag{41}$$

so that the dissipation inequality reduces to $\Phi_M = \boldsymbol{\sigma}':\dot{\varepsilon}^{vp} + k\dot{\gamma}_{vp} + \boldsymbol{Y}:\dot{\boldsymbol{D}} \geq 0$, with a "*damaged effective stress*" expressed as:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p_w \boldsymbol{B}(\boldsymbol{D}) \tag{42}$$

Now the objective is to express a viscoplastic potential of the form $\varphi_2^* = \varphi_2^*(\boldsymbol{\sigma}', k/\boldsymbol{D}, \gamma_{vp})$. Using the same approach as Pellet et al. (2005), the following expression is adopted:

$$\varphi_2^*(\boldsymbol{\sigma}', k/\boldsymbol{D}, \gamma_{vp}) = \varphi^*_{\mathscr{Q}}(\tilde{\boldsymbol{\sigma}}, k/\gamma_{vp}) = \frac{K}{N+1}\left\langle \frac{\tilde{q} + \alpha\tilde{p} - k}{K} \right\rangle^{N+1} \gamma_{vp}{}^{-N/M} \tag{43}$$

$\tilde{\boldsymbol{\sigma}}$ is a "*hydro-mechanical damaged effective stress*" accounting for multiple couplings:

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{M}:\boldsymbol{\sigma}' \tag{44}$$

This "*double effective stress*" combines the thermodynamic force-like variable $\boldsymbol{\sigma}'$ (associated to viscoplastic deformation) with the thermodynamic force-like variable $\tilde{\boldsymbol{\sigma}}$ (similar to the effective stress used in Continuum Damage Mechanics). $\tilde{p}$ and $\tilde{q}$ are the mean stress and the equivalent Von Mises stress linked to this "*double effective stress*".

Following Lemaitre et al. (2010) or Corbedois & Sidoroff (1982), viscoplastic strain rate is computed from:

$$\dot{\varepsilon}^{vp} = \frac{\partial\varphi_2^*}{\partial\boldsymbol{\sigma}'} = \frac{\partial\varphi_2^*}{\partial\tilde{\boldsymbol{\sigma}}}:\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}'} \tag{45}$$

In other words:

$$\dot{\varepsilon}^{vp} = \left\langle \frac{\tilde{q} + \alpha\tilde{p} - k}{K\gamma_{vp}{}^{1/M}} \right\rangle^{N} \left(\frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}}{\tilde{q}} + \frac{\alpha}{3}(\boldsymbol{\delta} - \boldsymbol{D}) \right) \tag{46}$$

with $\tilde{\tilde{\boldsymbol{s}}} = \boldsymbol{H}\cdot\tilde{\boldsymbol{s}}\cdot\boldsymbol{H}$.

The cumulative viscoplastic strain rate is calculated from:

$$\dot{\gamma}_{vp} = \sqrt{\frac{2}{3}}\left\langle \frac{\tilde{q} + \alpha\tilde{p} - k}{K\gamma_{vp}{}^{1/M}} \right\rangle^{N} \left\| \frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}}{\tilde{q}} + \frac{\alpha}{3}(\boldsymbol{\delta} - \boldsymbol{D}) \right\| \tag{47}$$

The combination of Eqs.(58) and (59) yields:

$$\dot{\varepsilon}^{vp} = \sqrt{\frac{3}{2}}\dot{\gamma}_{vp}\, \frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}}{\tilde{q}} + \frac{\alpha}{3}(\boldsymbol{\delta} - \boldsymbol{D}) \Big/ \left\| \frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}}{\tilde{q}} + \frac{\alpha}{3}(\boldsymbol{\delta} - \boldsymbol{D}) \right\| \tag{48}$$

The viscoplastic strain rate tensor is characterised by its norm $\sqrt{3/2}\,\dot{\gamma}_{vp}$ and its direction following the normal tensor $3\tilde{\tilde{\boldsymbol{s}}}/2\tilde{q} + \alpha/3(\boldsymbol{\delta} - \boldsymbol{D}) \Big/ \left\| 3\tilde{\tilde{\boldsymbol{s}}}/2\tilde{q} + \alpha/3(\boldsymbol{\delta} - \boldsymbol{D}) \right\|$. It depends on the viscoplastic strains which are already produced, actual stresses and damage of the material.

This approach can be applied to build a model taking creep, damage effects and poro-mechanical couplings into account by resorting to the concepts of "*damaged effective stress*" and "*hydro-mechanical effective stress*".

The step-by-step approach described in this paragraph is summarized in Fig. 1.

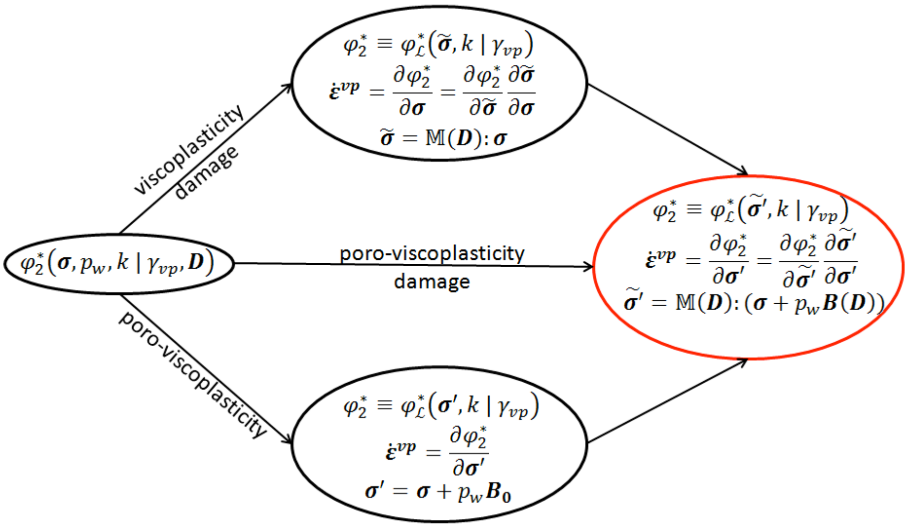

Fig 1. Construction of the viscoplastic dissipation potential based on the "*double effective stress*".

7. Conclusions

This paper shows how a few concepts and assumptions can be combined together in order to build a simple model, which can simulate the simultaneous occurrence of creep, damage and hydro-mechanical interactions in a saturated porous rock. The development is done within a general framework built upon the fundamental principles of thermodynamics. Under usual theoretical assumptions, the complete framework of a constitutive model requires the determination of two thermodynamic potentials: the free energy of the solid skeleton and the dissipation potential. However, these potentials are generally difficult to express within a physically realistic and thermodynamically consistent framework. In Continuum Damage Mechanics (Lemaitre et al., 2000), the state of stress of a monophasic solid material subject to damage and viscoplasticity is based on the concept of effective stress (called "*damaged effective stress*" in this paper). In poromechanics (Coussy, 2004), Biot's effective stress represents the hydro-mechanical couplings between the solid skeleton and the saturating pore fluid. A literature review shows that the coupling between damage, viscoplasticity and solid-fluid interactions still lacks a rigorous modeling framework. A simplified formulation is proposed in order to obtain a model with manageable complexity yet sufficiently complete to be able to simulate the main behavior trends. A step-by-step approach is adopted, by reviewing several base cases involving partial couplings only. The thermodynamic potentials are assumed to be partially uncoupled, and the irreversible rate of change of the viscoplastic porosity is related to the viscoplastic strain rate. The latter assumptions allow to reduce the dependence of the dissipation potential on total stress, pore pressure and damage to a dependence on a single thermodynamic force-like variable, called the "*double effective stress*". This "damaged hydro-mechanical effective stress" combines the "*damaged effective stress*" usually employed in Continuum

Damage Mechanics and the force-like variable associated to viscoplastic strains in poromechanics. The definition of the "*double effective stress"* is one of the key issues for the construction of damageable viscoplastic poromechanical models in a thermodynamically consistent framework.

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